

May 2013

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : 1581

Unique Paper Code : 222201

C

Name of the Paper : PHHT-203 : Mathematical Physics II

Name of the Course : B.Sc. (Hons.) PHYSICS

Semester : II

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all including Q. No. 1 which is compulsory.

1. Do any five of the following :

5×3=15

(i) Determine the order, degree and linearity of the differential equation :

$$\left(\frac{d^3 y}{dx^3}\right)^4 + \frac{d^2 y}{dx^2} + y \frac{dy}{dx} = 0.$$

(ii) If y_1 and y_2 are the two solutions of a homogeneous differential equation :

$$y'' + p(x)y' + q(x)y = 0,$$

prove that $c_1 y_1 + c_2 y_2$ is also a solution of the equation. (here, $y' = \frac{dy}{dx}$)

(iii) Check whether the following functions are linearly independent or not :

$$e^x, x e^x, e^{-x}.$$

P.T.O.

- (iv) Find the maximum or minimum values of the function :

$$f(x, y) = y^2 + 4xy + 3x^2 + x^3.$$

- (v) Check whether the following differential equation is exact, if so, solve it :

$$\left(e^x + \frac{2x}{y} \right) dx - \frac{x^2}{y^2} dy = 0.$$

- (vi) Prove the following property of Poisson Bracket :

$$[uv, w] = [u, w]v + u[v, w].$$

- (vii) Using Lagrange method of undetermined multipliers, find the maximum and minimum distance of the point (4, 6) from $x^2 + y^2 = 4$.

2. Solve the following differential equations :

(a) $\frac{dy}{dx} + xy = x^2 y^3$

(b) $(D^2 + 4)y = x \sin 2x.$

3. (a) Solve :

$$(D^2 - 4D + 4)y = x^3 + e^x + \cos 2x.$$

- (b) Solve :

$$\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 4y = x^2,$$

where $y(0) = 0$ and $y'(0) = \frac{1}{2}.$

4. (a) Solve :

$$(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin [\log (1+x)].$$

- (b) Using the method of undetermined coefficients, solve :

$$\frac{d^2 y}{dx^2} = 9x^2 + 2x - 1. \quad 9,6$$

5. (a) Solve the following simultaneous equations :

$$\frac{dy}{dx} + y = z + e^x$$

$$\frac{dz}{dx} + z = y + 2e^x.$$

- (b) Using the method of variation of parameters, solve :

$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2. \quad 7,8$$

6. (a) Derive Euler-Lagrange equation for a function $f(x, y, y')$.

- (b) Show that the path followed by a particle in vertical plane in sliding from one point to another in the absence of friction in the shortest time is a cycloid. 7,8

7. (a) A rectangular parallelopiped has a given surface area S . Determine its maximum volume using Lagrange Method of undetermined multiplier.

P.T.O.

(4)

(b) If

$$u = \frac{a^3}{x^2} + \frac{b^3}{y^2} + \frac{c^3}{z^2},$$

where $x + y + z = 1$, then prove that the stationary value of u is given by :

$$x = \frac{a}{a+b+c}, y = \frac{b}{a+b+c}, z = \frac{c}{a+b+c}.$$

8. (a) Define Lagrangian brackets and prove that :

(i) $\{q_j, q_k\} = 0$

(ii) $\{p_j, p_k\} = 0$ and

(iii) $\{q_j, p_k\} = \delta_{jk}$.

(b) A block of mass m , connected at one end of the spring (spring constant k) executing SHM (Fig. 1). Write the Lagrangian and hence determine the time period of oscillation.

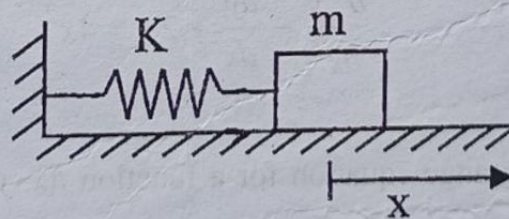


Fig. 1

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : 1610

26 NOV 2014

Unique Paper Code : 2221301

F-3

Name of the Paper : Mathematical Physics II

Name of the Course : B.Sc. (Hons.)

Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt Five questions in all.

Question No. 1 is compulsory.

1. Do any five :

5×3=15

(a) Prove that :

$$\int_{-L}^L \cos \frac{m\pi x}{L} \cos \frac{n\pi x}{L} dx = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases}$$

(b) State Dirichlet conditions for expansion of a periodic function in a Fourier series.

(c) Evaluate the value of $\Gamma[-5/2]$, given that $\Gamma[1/2] = \sqrt{\pi}$.

(d) Prove that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

for $n = 1, 2, 3, \dots$

P.T.O.

(e) Write appropriate partial differential equations applicable to the following :

(i) Small transverse vibrations of a taut, flexible violin string

(ii) Vibrations of the stretched circular membrane of a drum.

(f) Prove that :

$$P_n(-1) = (-1)^n.$$

(g) Show that Bessel's equation :

$$x^2 y'' + xy' + (x^2 - n^2)y = 0.$$

is transformed into :

$$x^2 y'' + xy' + (\lambda^2 x^2 - n^2)y = 0$$

if x is replaced by λx , where λ is a constant.

2. (a) Classify the following function as even, odd, or neither even nor odd and expand it in a Fourier series :

$$f(x) = \begin{cases} -x & -2 < x < 0 \\ x & 0 < x < 2 \end{cases} \quad \text{Period} = 4. \quad 10$$

- (b) Making use of Parseval's identity corresponding to the Fourier series of the above function, show that :

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots = \frac{\pi^4}{96}. \quad 5$$

3. (a) Solve Hermite's differential equation by Frobenius method :

$$y'' - 2xy' + 2ny = 0,$$

given n is an integer.

12

- (b) Discuss whether each of the points $x = +1, 0, -1$ is ordinary, regular singular point or irregular singular point of the differential equation :

$$(1 - x^2)y'' + x(1 - x)y' + (1 + x)y = 0.$$

3

4. (a) Obtain Rodrigue's formula for Legendre polynomials :

$$P_n(x) = \frac{1}{n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

Evaluate the value of $P_3(x)$ using Rodrigue's formula.

10

- (b) Expand :

$$f(x) = x^2 + 5x - 2$$

in a series of the form :

$$\sum_{n=0}^{\infty} a_n P_n(x).$$

$$[\text{Given : } P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{(3x^2 - 1)}{2}]$$

5

5. (a) Using generating function of Bessel's function verify that :

6

$$(i) \cos(x \sin \theta) = J_0(x) + 2J_2(x) \cos 2\theta + 2J_4(x) \cos 4\theta + \dots$$

$$(ii) \sin(x \sin \theta) = 2J_1(x) \sin \theta + 2J_3(x) \sin 3\theta + 2J_5(x) \sin 5\theta + \dots$$

P.T.O.

(b) Establish that :

$$\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}; \quad m, n > 0.$$

5

(c) Evaluate :

$$\int_0^{\infty} \sqrt[4]{x} e^{-\sqrt{x}} dx.$$

40

6. (a) Find Fourier series expansion of the output of a full wave rectifier.

8

(b) Show that :

$$J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x - x \cos x}{x} \right).$$

.7

7. Find the amplitude $u(\rho, \phi, t)$ of a vibrating circular membrane of radius 'a' satisfying the wave equation :

$$\nabla^2 u - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0.$$

Here 'v' is the phase velocity of the wave fixed by elastic constants.

15

9

This question paper contains 4 printed pages]

Roll No.

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S. No. of Question Paper : 935

08 MAY 2015

Unique Paper Code : 222201

E

Name of the Paper : PHHT-203 : Mathematical Physics II

Name of the Course : B.Sc. (Hons.) PHYSICS

Semester : II

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all. Question No. 1 is compulsory.

Attempt four questions from the rest of the question.

1. Do any five of the following :

5×3=15

(a) Determine the order, degree and linearity of the following differential equations :

$$\left(\frac{d^2x}{dy^2}\right)^2 + \frac{d^2x}{dy^2} + 4\frac{dx}{dy} = 0.$$

(b) What is Wronskian ? Calculate the Wronskian of the functions x, x^2, x^3 .

P.T.O.

(c) Two solutions of $y'' - 2y' + y = 0$ are e^x and xe^x . Is the general solution of the given equation $y = C_1 e^x + C_2 x e^x$? Discuss.

(d) Write the Euler-Lagrange's equation and explain the terms involved.

(e) Solve the differential equation :

$$(2xy + 1)dx + (x^2 + 4y)dy = 0.$$

(f) Find the shortest distance of the circle $x^2 + y^2 = 1$ from the point (2, 3).

(g) Define holonomic and non-holonomic constraints. Give *one* example each.

2. Solve the following differential equations :

(a) $(x^2 - ay)dx + (y^2 - ax)dy = 0$

(b) $\sec^2 y \, dy / dx + x \tan y = x^3$

(c) $\frac{dy}{dx} + y = \sin x; y(\pi) = 1.$

5,5,5

3. Solve :

(a) $(D - 2)^2 y = 8(e^x + \sin 2x + x^2)$

(b) $100 \frac{d^2 N}{dt^2} - 20 \frac{dN}{dt} + N = 0.$

9,6

4. (a) Solve :

$$x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = x \log x.$$

- (b) Using the method of undetermined coefficients, solve :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 4x + 10 \sin x. \quad 9,6$$

5. (a) Solve the coupled differential equations :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2 \cos t - 7 \sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4 \cos t - 3 \sin t.$$

- (b) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{3x}. \quad 8,7$$

6. (a) Derive Euler-Lagrange's equation for a function $f(x, y, y')$.

- (b) Show that the path followed by a particle in vertical plane in sliding from one point to another in the absence of friction in the shortest time is a cycloid. 7,8

7. (a) Using Lagrange's method of undetermined multipliers, find the largest product of x, y, z , when these are related as $x^2 + y^2 + z^2 = 9$.

- (b) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \quad 7,8$$

P.T.O.

8. (a) Define Lagrangian Bracket and prove that :

$$(i) \quad [q_j, q_k] = 0$$

$$(ii) \quad [p_j, p_k] = 0$$

$$(iii) \quad [q_j, p_k] = \delta_{jk}.$$

(b) A simple pendulum of mass m and length l executing SHM. Write the Lagrangian and hence determine the time period of small oscillation. 7,8

[This question paper contains 2 printed pages.]

Sl. No. : 2070 GC-3 Your Roll No.....
Unique Paper Code : 32221304
Name of Paper : Mathematical Physics-II
Name of the Course : B.Sc (Hons) Physics
Scheme of Examination : CBCS Part-III (Sem-III)
Duration : 3 hours
Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No at the top of your Question Paper.
2. Question no 1 is compulsory. Attempt five questions in all.
3. In the question paper $y \equiv y(x)$, $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$.

1. Attempt any five questions

- a) Find the indicial equation and its roots for the given differential equation

$$x^2 y'' + 3xy' + 4xy = 0$$

- b) Evaluate the value of $\frac{\Gamma(3)\Gamma(3/2)}{\Gamma(9/2)}$

- c) Find the value of $P_n(-1)$.

- d) State the Dirichlet's conditions.

- e) Express $f(x)$ as a linear combination of Legendre Polynomials where

$$f(x) = 5x^3 + 2x^2 - 7x + 4$$

- f) Given that $f(x) = f(x+2\pi)$ and if

$$f(x) = |x| \text{ if } -\pi < x < \pi$$

Graphically represent the function in the interval $(-\pi, \pi)$

- g) Evaluate $\int_0^2 x^3 \sqrt{8-x^3} dx$

$$5 \times 3 = 15$$

2. A rectangular homogeneous membrane of length a and breadth b is fixed at the boundaries. The membrane is free to oscillate vertically with respect to its plane. Initially, the membrane was at rest and its initial position is given as $f(x, y)$. Find the displacement at any time 't' and at position (x, y) .

15

P.T.O.

3. Find the Fourier series expansion of the function

$$f(x) = \begin{cases} -kif & -\pi < x < 0 \\ kif & 0 < x < \pi \end{cases} \text{ and } f(x + 2\pi) = f(x)$$

Hence find the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

12+3=15

4. Prove that

$$\int_{-1}^1 P_m(x) P_n(x) dx = \frac{2}{2n+1} \delta_{mn}$$

12x7.5=15

Q5.a) Solve the given differential equation

$$y'' - 2xy' + 2\lambda y = 0$$

λ is a real constant. For $\lambda=n=0,1,2,3\ldots$, show that one of the solution is a polynomial of order n . Name the polynomial.

b) Show that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin(x)$$

12+3=15

6. a) For Legendre polynomial $P_n(x)$, prove that

$$nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$$

b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

c) Show that $\frac{d}{dx} \{x^n J_n(x)\} = x^n J_{n-1}(x)$

3x5=15

7. Solve the differential equation

15

$$x^2 y'' + xy' + (x^2 - 4)y = 0$$

[This question paper contains 3 printed pages.]

30 MAY 2016

Sr. No. of Question Paper : 5776 F Your Roll No.....

Unique Paper Code : 222201

Name of the Paper : PHHT-203: Mathematical Physics II

Name of the Course : B. Sc. (Hons.) PHYSICS

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all. Question No. 1 is compulsory.
3. Attempt **four** questions from the rest of the paper

1. Attempt any five of the following: (3x5=15)

(a) Determine the order, degree and linearity of the following differential equations:

(i) $\left(\frac{d^2x}{dy^2}\right)^2 + \frac{d^2x}{dy^2} + 4\frac{dx}{dy} = 0$

(ii) $\left(\frac{d^2y}{dx^2}\right)^{3/2} + y^{1/2} = x$

(b) What is Wronskian? Calculate the Wronskian of the functions:

e^x, xe^x, e^{-x}

(c) If y_1 and y_2 are the two solutions of a homogeneous differential equation

$y'' + p(x)y' + q(x)y = 0$

Prove that $c_1y_1 + c_2y_2$ is also a solution of the equation. (Here

$y' = \frac{dy}{dx}$).

9/8

(d) Write the Euler-Lagrange's equation and explain the terms involved.

(e) Solve the differential equation:

$$(2xy+1)dx + (x^2 + 4y)dy = 0$$

(f) Find the shortest distance of the circle $x^2 + y^2 = 1$ from the point (2,3).

(g) A block of mass m connected at one end of the spring of spring constant k is executing SHM. Write the Lagrangian for the system.

2. (a) Solve $\frac{d^2 y}{dx^2} = [1 - (\frac{dy}{dx})^2]^{1/2}$ (3)

(b) Prove $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$ (5)

(c) For the differential equation whose solutions are $y_1 = e^{-x} \cos x, y_2 = e^{-x} \sin x$, show that they are linearly independent. (4)

(d) Solve $\frac{d^2 y}{dx^2} = 9x^2 - 1$ (3)

3. (a) Solve: $(D^2 - 2D)y = e^{2x} \sin x$ (4)

(b) Using the method of undetermined coefficient, solve

$$(D^2 + 4)y = 4 \sin 2x + 2e^{2x}. \quad (5)$$

(c) Solve $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x \log x$. (6)

4. Solve the following differential equations:

(a) $\frac{d^2 y}{dx^2} + xy = xy^3$ (5)

(b) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \sin x$ (4)

- (C) Using the method of variation of parameter, solve

$$(D^2 + 1)y = (x + \cot x) \quad (6)$$

5. (a) Find the maximum and the minimum distance from the origin to the curve

$$5x^2 + 6xy + 5y^2 = 8. \quad (6)$$

- (b) Determine the curve joining two given points which produce the least area when it is revolved about the x-axis lying in the plane of the curve.

(9)

6. (a) Solve the coupled differential equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t \quad (8)$$

- (b) Derive Euler-Lagrange Equation for a function $f(x, y, y')$. (7)

7. (a) Define Poisson Bracket's and prove that

$$(i) [q_j, q_k] = 0$$

$$(ii) [p_j, p_k] = 0$$

$$(iii) [q_j, p_k] = \delta_{jk} \quad (7)$$

- (b) A simple pendulum of pointmass m and length l is executing SHM. Write the Lagrangian and hence determine the time period for small oscillations (8)

(300)

This question paper contains 4+2 printed pages]

Roll No.

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S. No. of Question Paper : 860

6

Unique Paper Code : 222201

G

Name of the Paper : Mathematical Physics-II

Name of the Course : B.Sc. (Hons.) Physics

Semester : II

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.

Question No. 1 is compulsory.

I. Do any five questions :

5×3=15

(a) Determine the order, degree and linearity of the differential equation :

$$\left(\frac{d^2y}{dx^2}\right)^2 + \frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0.$$

P.T.O.

4. (a) Use the method of undetermined coefficients to solve the differential equation :

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = x^2 + \cos 2x.$$

- (b) Solve for 'y' :

7,8

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{(\log(x))^2}{x}.$$

5. (a) Solve the differential equation using method of variation of parameters :

$$\frac{d^2 y}{dx^2} + a^2 y = \operatorname{cosec} ax.$$

- (b) Find the value of y and z from the following coupled differential equations :

7,8

$$\frac{dy}{dx} + y = z + e^x$$

$$\frac{dz}{dx} + z = y + 2e^x.$$

6. (a) Derive the Euler-Lagrange's equation for a function $f(x, y, y')$.

(b) Prove that the shortest path between two points in a plane is the straight line joining the points. 8,7

7. (a) Using the Lagrange's method of undetermined multiplier, find out the largest product of x, y and z when they are constrained by the relation :

$$x^2 + y^2 + z^2 = 9.$$

(b) Find the shortest distance between the origin and the curve defined by the equation : 8,7

$$5x^2 + 5y^2 + 6xy = 8.$$

8. (a) A simple pendulum of mass ' m ' and length ' l ' is executing simple harmonic motion. Write down the Lagrangian and hence determine the time period of small oscillations.

P.T.O.

(i) Define Poisson Bracket and show that :-

$$(i) \quad [X, YZ] = Y[X, Z] + [X, Y]Z$$

$$(ii) \quad [q_j, H] = \dot{p}_j$$

$$(iii) \quad [q_j, p_k] = \delta_{jk}$$

Here H is the Hamiltonian.

This question paper contains 4 printed pages.

Your Roll No.

Sl. No. of Ques. Paper : 104 I
Unique Paper Code : 32221301
Name of Paper : Mathematical Physics – II
Name of Course : B.Sc. (Hons.) Physics
Semester : III
Duration : 3 hours
Maximum Marks : 75

03 DEC 2018

(Write your Roll No. on the top immediately
on receipt of this question paper.)

Question No. 1 is compulsory.
Attempt five questions in all.

1. Attempt any five questions :

(a) Identify the singularities of the equation :

$$x^2(1-x^2)y'' + \frac{2}{x}y' + 4y = 0.$$

(b) Show that :

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

(c) Prove that :

$$P'_n(1) = \frac{n(n+1)}{2}.$$

(d) Evaluate

$$\Gamma\left(\frac{-3}{2}\right).$$

P.T.O.

(e) State the Dirichlet conditions.

(f) Prove that :

$$\int_{-L}^L \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases}$$

where m and n can assume any of the values $1, 2, 3, \dots$

(g) Show that $\beta(a, b) = \beta(a+1, b) + \beta(a, b+1)$. 5×3=15

2. Find the Fourier series of periodic function defined by

$$f(x) = x^2 \quad \text{for } -\pi < x < \pi$$

and hence show that :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \quad 15$$

3. Using Frobenius method, obtain the solution of the following equation about $x = 0$:

$$(x-1)y'' + (3x-1)y' + y = 0.$$

(a) Evaluate :

$$\int_0^1 x^3 J_0(x) dx$$

and express it in terms of $J_0(x)$ and $J_1(x)$. 10

4. (a) Expand $x^4 - 3x^2 + x$ in a series of the form

$$\sum_{k=0}^{\infty} A_k P_k(x)$$

for at least $k = 0, 1$ and 2 .

(b) Prove that :

$$(2n+1) P_n(x) = P_{n+1}(x) - P_{n-1}(x). \quad 10,5$$

5. (a) Prove that :

$$\int_0^1 x J_n(ax) J_n(bx) dx = \frac{1}{2} J_{n+1}^2(x) \delta_{ab}$$

where $J_n(a) = J_n(b) = 0$.

(b) Prove that :

$$nJ_n(x) + xJ'_n(x) = xJ_{n-1}(x). \quad 10,5$$

6. (a) Solve :

$$\int_0^1 \frac{dx}{\sqrt{-\ln x}}$$

using Gamma function.

(b) Show that :

$$\int_0^1 x^{m-1} (1-x^2)^{n-1} dx = \frac{1}{2} \beta\left(\frac{m}{2}, n\right).$$

(c) Find out the roots of indicial equation for the Laguerre's differential equation :

$$xy'' + (1-x)y' + \lambda y = 0$$

where λ is a constant.

7. Find the solution of 1-dimensional wave equation :

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2}; \quad 0 \leq x \leq 1, \quad t > 0$$

under the boundary conditions $U(0, t) = U(1, t) = 0$ and initial conditions :

$$U(x, 0) = \begin{cases} x & 0 \leq x \leq \frac{\ell}{2} \\ \ell - x & \frac{\ell}{2} \leq x \leq \ell \end{cases} \quad \text{and} \quad \left. \frac{\partial U}{\partial t} \right|_{t=0} = 0. \quad 15$$